

## BINOMIAL EXPANSION MARKING SCHEME

1.	$1^8 + 1^7\left(\frac{1}{2}x\right) + 1^6\left(\frac{1}{2}x\right)^2 + 1^5\left(\frac{1}{2}x\right)^3$ $1 + 8\left(\frac{1}{2}x\right) + 28\left(\frac{1}{4}x^2\right) + 56\left(\frac{1}{8}x^3\right)$ $1 + 4x + 7x^2 + 7x^3$ $1 + \frac{1}{2}(0.1) = 1.05$ $1 + 0.4 + 0.07 + 0.007$ $= 1.477$ $= 1.48$ <p style="text-align: right;"><b>1989Q13</b></p>	5M	
2.	$p^8 + 8p^7q + 28p^6q^2 + 56p^5q^3 + 56p^5q^3$ $(9.99)^8 = (p+q)^8$ $(9+0.99)^8 = (p+q)^8$ $9^8 + 8 \cdot 9^7 \cdot 0.99 + 28 \cdot 9^6 \cdot 0.99^2 + 56 \cdot 9^5 \cdot 0.99^3$ $43046721 + 38263752 \cdot 0.99 + 145842 \cdot 29.07 + 3208530.396$ $= 99103233.46$ $= 99103200$ <p style="text-align: right;"><b>1990Q16</b></p>	4M	
3.	$15 - 14(2x) + 13(2x)^2 - 12(2x)^3 + 1(2x)^4 - (2x)^5$ $1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$ $(1 - 2x)^5 = (1 - 0.02)^5$ $2x = 0.02$ $x = 0.01$ $; x = 0.01; 0.90392 \quad (\text{to 5d.p})$ <p style="text-align: right;"><b>1991Q16</b></p>	3M	
4.	$1 - 4x + 7x^2 - 7x^3$ $(1 - \frac{1}{2}x)^8 = (1 - 0.02)^8$ $\frac{1}{2}x = 0.02$ $x = 0.04$ $1 - 4(0.04) + 7(0.04)^2 - 7(0.04)^3$ $; 0.850752$ <p style="text-align: right;"><b>1992Q3</b></p>	3M	
5.	$(2 + 0.002)^4$ $2^4 + 2^3(0.002) + 2^2(0.002)^2 + 2(0.002)^3 + 0.002^4$ $16 + 0.064 + 0.000096 + 0.000000064 + 0.000000000016$ $= 16.0609606; 16.0641(\text{to 4 d.p})$ <p style="text-align: right;"><b>1993Q7</b></p>	3M	
6.	$1^{10} + 1^9(2x) + 18(2x)^2 + 17(2x)^3$	4M	
	$1 + 20x + 180x^2 + 960x^3$ $(1+2x)^{10} = (1-0.05)^{10}$ $2x = -0.05 \quad x = -0.025$ $1 + 20(0.025) + 180(-0.025)^2 + 960(-0.025)^3$ $1 - 0.5 + 0.1125 - 0.015$ $= 0.598 \quad (\text{to 3d.p})$ <p style="text-align: right;"><b>1994Q5</b></p>		
7.	$(1 + ta)^5 = 1 + 5a + 10a^2 + 10a^3 + 5a^4 + a^5$ $(1 - 0.2)^5 = 1 - 5(0.2) + 10(-0.2)^2 + 10(-0.2)^3 + (-0.2)^4 + (-0.2)^5$ $1 - 1 + 4 - 0.08 + 0.008 - 0.00032$ $= 0.40800 - 0.8032 = 0.00032$ $= 0.40800 - 0.8032 = 0.32768$ $= 0.3277$ <p style="text-align: right;"><b>1996Q12</b></p>	B1 M1 A1 3M	
8.	$(1 - 3x)^5$ $= 1 + 5(-3x) + 10(-3x)^2 + 10(-3x)^3 + 5(-3x)^4 + (-3x)^5$ $= 1 - 15x + 90x^2 - 270x^3 + 540x^4 - 243x^5$ $= 3x = 0.03 \quad \text{or } x = 0.1$ $(0.97)^5$ $= 1 - 15(0.01) + 90(0.01)^2 - 270(0.01)^3 + 540(0.01)^4 - 243(0.01)^5$ $= 1.0 \cdot 15 + 0.009 - 0.00027$ $= 0.85873$ $= 0.8587 \quad \text{to 4 d.p}$ <p style="text-align: right;"><b>1997Q9</b></p>	M1 A1 B1  M1 A1 5M	
9.	$1 + 6x + 15x^2 + 20x^3 + 6x^4 + x^5$ $1 + 6(0.03) + 15(0.03)^2 + 20(0.03)^3 + 6(0.03)^4 + (0.03)^5$ $= 1 + 0.18 + 0.135 + 0.0054 + 0.000162 + 0.00000243$ $= 1.19404$ $= 1.194$ <p style="text-align: right;"><b>1998Q13</b></p>	B1 M1  2M	
10.	$(0.96)^5 = (1.04)^{-5}$ $= 1 + 5(0.04) + 10(-0.04)^2 + 10(-0.04)^3 + 5(-0.04)^4 + (-0.04)^5$ $= 1 - 0.2 + 0.016 - 0.00064 + 0.000016 - 0.000001024$ $= 0.81536$ $(0.8153728 \text{ or } 0.8153726976)$ $= 0.8154 \quad (\text{to 4 s.f})$ <p style="text-align: right;"><b>1999Q10</b></p>		
11.	$(1 + x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$ $(1 + 0.04)^5 = 1 + 5(0.04) + 10(0.04)^2 + 10(0.04)^3 + 5(0.04)^4 + (0.04)^5$ $= 1 + 0.2 + 0.16 + 0.0256 + 0.0032 + 0.0001024$ $= 1.2166528$ $= 1.2167 \quad (4 \text{ d.p})$ <p style="text-align: right;"><b>2000Q13</b></p>	B1 M1 M1 A1 4M	
12.	$(2+x)^3 = 2^3 + 2^2 \cdot 3x + 2 \cdot 3x^2 + x^3$ $= 8 + 12x + 6x^2 + x^3$		

	$= 32 + 80x + 80x^2 + 40x^3$ $(2.03)^5 = 32 + 80 \times 0.03 + 80 \times (0.03)^2 + 40 \times (0.03)^3$ $= 32 + 2.4 + 0.072 + 0.00108$ $= 34.47308$ $= 34.47$ (significant figures) <b>2001Q10</b>	
13.	$(a-b)^6 = a^6 - 6a^5b + 20a^3b^3 + 20a^3b^4 - 6ab^3 + b^6$ $1.98 = 2 - 0.02$ $\log 6 = 2^6 6(2)^5(0.02) + 15(2)^4(0.03)^2$ $= 64 - 3.84 + 0.096 = 60.256$ <b>2002Q9</b>	B1 B1 M1 A1
14.	a) $(8-x)^2 = 2^6 - 6 \cdot 2^5x + 15 \cdot 2^4x^2 - 20 \cdot 2^3x^3 + 15 \cdot 2^2x^4 - 6 \cdot 2x^5$ $= 64 - 192x + 240x^2 - 160x^3 + 60x^4 - 12x^5 + x^6$ b) $1.993 = (2 - 0.01)^6$ $= 64 - 192x + 240x^3 - 160x^3$ $= 64 - 1.92 + 0.24$ $= 62.104$ <b>2003Q11</b>	M1 A1 M1 A1 4 M
15.	(1+x) <sup>5</sup> = 1 + 5x + 10x <sup>2</sup> + 10x <sup>3</sup> + 5x <sup>4</sup> + x <sup>5</sup> b). x = 0.02 $1 + 5(-0.02) + 10(-0.02)^2$ $= 1 - 0.1 + 0.04$ $= 0.904$ <b>2004Q8</b>	
16.	$(3x-y)^4 = (3x)^4y^0 + (3x)^3y^1 + (3x)^2y^2 + (3x)^1y^3 + (3x)^0y^4$ $= 81x^4 + 27x^3y + 9x^2y^2 + 3xy^3 + y^4$ With coeff. $(3x-y)^4 = 81x^4 - 4x \cdot 27x^3y + 6x \cdot 9x^2y^2 - 4x \cdot 3xy^3 + y^4$ $81x^4 - 108x^3y + 54x^2y^2 - 36xy^3 + y^4$ x=2 and y=0.2 $(6-0.2)^4 = 81x^2$ $= 108x^2 \times 0.2x \times 54x^2 \times 0.2^2$ $= 205.2$ <b>2005Q13</b>	B1 M1 M1 A1
17.	$(2 + \frac{1}{2})^5 + 5(2^4)(\frac{1}{\sqrt{2}}) + 10(2^3)(\frac{1}{\sqrt{2}})^2$	M1

	$= 10(2^2)(\frac{1}{\sqrt{2}})^3 + 5(2)(\frac{1}{\sqrt{2}})^4 + (\frac{1}{2})^5$ $(2 - \frac{1}{2})^5 = 25 - 5(2^4)(\frac{1}{2}) + 10(2^3)(\frac{1}{\sqrt{2}})^5$ $- 10(2^2)(\frac{1}{\sqrt{2}})^3 + 5(2)(\frac{1}{\sqrt{2}})^4 - (\frac{1}{\sqrt{2}})^5$ $= 2[2^5 + 10(2^3)(\frac{1}{\sqrt{2}})^2 + 5(2)(\frac{1}{\sqrt{2}})^4]$ $= 64 + 80 + 5$ $= 149$ <b>2006Q11</b>	M1 M1 A1 4 M
18.	a). $(1 + \frac{1}{2}x)^5 = 1^5 \cdot 1.1(1.2x)^0 + 5 \cdot 1^4(\frac{1}{2}x)^1 + 10 \cdot 1^3(\frac{1}{2}x)^2 + 10 \cdot 1^2(\frac{1}{2}x)^3 + 5 \cdot 1^1(\frac{1}{2}x)^4 + 1 \cdot 1^0(\frac{1}{2}x)^5$ $= 1 + \frac{5}{2}x + \frac{5}{2}x^2 + \frac{5}{4}x^3 + \frac{5}{16}x^4 + \frac{1}{32}x^5$ b). $1 \frac{1}{20} = 1 + \frac{1}{20} = \frac{1}{2}x = \frac{1}{20}x = \frac{1}{10}$ $(1 \frac{1}{20})^5 = 1 + \frac{5}{2}x \frac{1}{10} + \frac{5}{2}x \frac{1}{100}$ $= 1 + \frac{5}{20} + \frac{5}{200} = 1 \frac{11}{40}$ <b>2007Q4</b>	M1 A1 M1 A1 4 m
19.	a). $(10 + \frac{2}{x})^5 = 10^5 + 10^4(\frac{2}{x}) + 5 \cdot 10^3(\frac{2}{x})^2 + 5 \cdot 10^2(\frac{2}{x})^3 + 5 \cdot 10(\frac{2}{x})^4 + (\frac{2}{x})^5$ $= 100000 + \frac{100000}{x} + \frac{40000}{x^2} + \frac{2000}{x^3} + \frac{800}{x^4} + \frac{32}{x^5}$ b). $(14^5) = 10x^2/x^5 \quad 2/x = 4$ $x = 2/3 = \frac{1}{2}$ $\frac{100000}{\frac{1}{2}} + \frac{100000}{(\frac{1}{2})^2} + \frac{2000}{(\frac{1}{2})^3} + \frac{800}{(\frac{1}{2})^4} + \frac{32}{(\frac{1}{2})^5}$ $100000 + 20000 + 16000 + 64000 + 12800 + 1024$ $= 537824$ <b>2008Q8</b>	M1 M1 M1 A1 4 M
20.	$(2-x)^7 = 2^7 - 7 \cdot 2^6x + 21 \cdot 2^5x^2 - 35 \cdot 2^4x^3 + 35 \cdot 2^3x^4 - 21 \cdot 2^2x^5 + 7 \cdot 2^1x^6 - x^7$ $= 128 - 448x + 672x^2 - 560x^3 + 280x^4 - 112x^5 + 56x^6 - x^7$	B1 A1

	$84x^5+14x^6-x^7$ b). $91.97^7 = (2.0.03)^7$ $= 128-448(0.03)+672(0.003)^2 -$ $560(0.03)^3$ $= 128-13.44+0.6048-0.01512$ $= 115.14968$ $= 115.1497$ <b>2009Q8</b>	M1 A1 4 M
21.	(a) $(2 - x)^5 = 25 - 5(-2)^4 x + 10(2)^3$ $x^2 - 10(2)^2 x^3 + 5(2)^4 - (x^5)$ $= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$  (b) $(2-0.2)^5 - 32 - 80(0.2) +$ $80(0.2)^2 - 40(0.2)^3$ $= 18.88$ <b>2010Q12</b>	
22.	$(a + \frac{1}{2})^4$ $= a^4 + 4a^3 (\frac{1}{2}) + 6a^2 (\frac{1}{2})^2 +$ $4a(\frac{1}{2})^3 + (\frac{1}{2})^4$ $= a^4 + 2a^3 + \frac{2}{3}a + \frac{1}{2}a + \frac{1}{16}$  $(a - \frac{1}{2})^4$ $= a^4 + 4a^3 (-\frac{1}{2}) + 6a^2(-\frac{1}{2})^2 +$ $4a(-\frac{1}{2})^3 + (-\frac{1}{2})^4$ $= a^4 - 2a^3 + \frac{3}{2}a^2 - \frac{1}{2}a + \frac{1}{16}$ $(a + \frac{1}{2})^4 (a - \frac{1}{2})^4 = 2a^4 + 3a^2 + \frac{1}{8}$ <b>2011Q11</b>	M1 M1 A1 3
23.	(a) $(1 + x)^7 = 1 + 7x + 21x^2 + 35x^3$  (b) $(0.94)^7 = [1 + (-0.06)]^7$ $= 1 + 7 \times (-0.06) + 21 \times$ $(-0.06)^2 + 35 \times (-0.06)^3$ $= 1 - 0.42 + 0.0756 - 0.00756$ $= 0.64804$	M1 A1 2

