MATRICES.

SECTION A.

1) Given that $\mathbf{P} = \begin{pmatrix} x & -1 \\ 15 & -x \end{pmatrix}$ and that $\mathbf{P}^2 = \mathbf{I}$, find x.

$$P = \begin{pmatrix} x & -1 \\ 15 & -x \end{pmatrix}$$

$$P^2 = \text{Identity}$$

$$P^2 = \begin{pmatrix} x & -1 \\ 15 & -x \end{pmatrix}\begin{pmatrix} x & -1 \\ 15 & -x \end{pmatrix}$$

$$= \begin{pmatrix} x^2 - 15 & -x + x \\ 15x - 15x & -15 + x^2 \end{pmatrix}$$

$$= \begin{pmatrix} x^2 - 15 & 0 \\ 0 & -15 + x^2 \end{pmatrix}$$

$$\text{Since } I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$x^2 - 15 = 1$$

$$x^2 = 16$$

$$x = \sqrt{14}$$

$$x = \pm 4.$$

2) Determine the value of x if the matrix $\mathbf{A} = \begin{pmatrix} 2x & -8 \\ -4 & x \end{pmatrix}$ has no inverse.

$$A = \begin{pmatrix} 2x & -8 \\ -4 & x \end{pmatrix}$$

$$\text{Determinant} = 0$$

$$2x^2 - 32 = 0$$

$$2x^2 = 32$$

$$x^2 = 16$$

$$x = \sqrt{14}$$

$$x = \pm 4.$$

3) Find the value of x of which the matrix $\mathbf{B} = \begin{pmatrix} x+1 & 5 \\ 4 & x \end{pmatrix}$ is singular.

$$B = \begin{pmatrix} x+1 & 5 \\ 4 & x \end{pmatrix}$$

$$\text{Determinant} = 0$$

$$x(x+1) - 20 = 0$$

$$x^2 + x - 20 = 0$$

Using quadratic formula;

$$x = \frac{-1 \pm \sqrt{1 + 80}}{2}$$

$$= \frac{-1 \pm 9}{2}$$

$$x = -5$$

or

$$x = -4.$$

4) Given that $\mathbf{M} = \begin{pmatrix} 4 & 5 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}$, find $\mathbf{M^{-1}N}$

$$\mathbf{M} = \begin{pmatrix} 4 & 5 \\ 2 & 3 \end{pmatrix} \text{ and } \mathbf{N} = \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}$$

$$\mathbf{M^{-1}} = ?$$

Det M;

$$= (4 \times 3) - (2 \times 5)$$

$$= 12 - 10$$

$$= 2$$

$$\mathbf{M^{-1}} = \frac{1}{2}\begin{pmatrix} 3 & -5 \\ -2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1.5 & -2.5 \\ -1 & 2 \end{pmatrix}$$

$$\mathbf{M^{-1}N} = \begin{pmatrix} 1.5 & -2.5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} (-3 - 2.5) & (4.5 + 2.5) \\ 4 & (-3 - 2) \end{pmatrix}$$

$$= \begin{pmatrix} -5.5 & 6.5 \\ 4 & -5 \end{pmatrix}.$$

5) Find the value of y given that the matrix $\mathbf{T} = \begin{pmatrix} y + 7 & 4 \\ -3 & y \end{pmatrix}$ is singular.

$$\mathbf{T} = \begin{pmatrix} y + 7 & 4 \\ -3 & y \end{pmatrix}$$

$$\text{Determinant} = 0$$

$$y(y + 7) + 12 = 0$$

$$y^2 + 7y + 12 = 0$$

using quadratic formula;

$$y = \frac{-7 \pm \sqrt{29 - 48}}{2}$$

$$= \frac{-7 \pm 1}{2}$$

$$y = -4$$

or

$$y = -3.$$

6) Given that A and B are matrices $\mathbf{A} = \begin{pmatrix} 0 & 2 \\ 0 & x - 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} x & 2 \\ 1 & -2 \end{pmatrix}$, find x if A + B is singular.

$$\mathbf{A} = \begin{pmatrix} 0 & 2 \\ 0 & x - 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} x & 2 \\ 1 & -2 \end{pmatrix}$$

A + B;

$$= \begin{pmatrix} 0 & 2 \\ 0 & x - 1 \end{pmatrix} + \begin{pmatrix} x & 2 \\ 1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} x & 4 \\ 1 & x - 3 \end{pmatrix}$$

For singular matrix;

$$\text{Det} = 0$$

$$x(x - 3) - 4 = 0$$

$$x^2 - 3x - 4 = 0$$

Using quadratic formula;

$$x = \frac{3 \pm \sqrt{9 + 16}}{2}$$

$$= \frac{3 \pm 5}{2}$$

$$x = 4$$

or

$$x = -1.$$

7) Given that $\mathbf{P} = \begin{pmatrix} 1 & k \\ 3 & 3 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} -3 & 5 \\ 1 & -2 \end{pmatrix}$, find k if the determinant of **PQ** is 9.

$$\mathbf{P} = \begin{pmatrix} 1 & k \\ 3 & 3 \end{pmatrix} \text{ and } \mathbf{Q} = \begin{pmatrix} -3 & 5 \\ 1 & -2 \end{pmatrix}$$

$$\mathbf{PQ} = \begin{pmatrix} 1 & k \\ 3 & 3 \end{pmatrix}\begin{pmatrix} -3 & 5 \\ 1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} -3+k & 5-2k \\ 0 & 9 \end{pmatrix}$$

But Det of PQ = 9

Therefore;

$$9(-3+k) = 9$$

$$-3+k = 1$$

$$k = 4.$$

8) Given that $\mathbf{P} = \begin{pmatrix} k & 4 \\ 3 & 2 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, find k if $|\mathbf{PQ}| = 4$

$$\mathbf{P} = \begin{pmatrix} k & 4 \\ 3 & 2 \end{pmatrix} \text{ and } \mathbf{Q} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$\mathbf{PQ} = \begin{pmatrix} k & 4 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} k+12 & k+16 \\ 9 & 14 \end{pmatrix}$$

But Det of PQ = 4

Therefore;

$$14(k+12) - 9(k+16) = 4$$

$$14k + 168 - 9k - 144 = 4$$

$$5k + 24 = 4$$

$$5k = -20$$

$$k = -4.$$

9) Find the value of x given that the matrix $\mathbf{R} = \begin{pmatrix} x & 6 \\ 4 & x-2 \end{pmatrix}$ is singular.

For singulsr matrix;

Det = 0

$$x(x-2) - 24 = 0$$

$$x^2 - 2x - 24 = 0$$

Using quadratic formula;

$$x = \frac{2 \pm \sqrt{4+96}}{2}$$

$$= \frac{2 \pm 10}{2}$$

$$x = 6$$

or

$$x = -4.$$

10) Given that $\mathbf{A} = \begin{pmatrix} x & 0 \\ 5 & y \end{pmatrix}$, find the value of x and y if $\mathbf{A^2} = \mathbf{I}$

$$\mathbf{A^2 = Identity}$$
$$\mathbf{A^2 = \begin{pmatrix} x & 0 \\ 5 & y \end{pmatrix}\begin{pmatrix} x & 0 \\ 5 & y \end{pmatrix}}$$
$$\mathbf{= \begin{pmatrix} x^2 & 0 \\ 5x + 5y & y^2 \end{pmatrix}}$$
$$\mathbf{I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$
$$\mathbf{\begin{pmatrix} x^2 & 0 \\ 5x + 5y & y^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$
$$\mathbf{x^2 + y^2 = 1 + 1}$$
$$\mathbf{x^2 + y^2 = 2 \ldots\ldots eq.1}$$
$$\mathbf{5x + 5y = 0}$$
$$\mathbf{5x = -5y}$$
$$\mathbf{x = -y \ldots. eq.2}$$
solving simulteneousely;
$$\mathbf{(-y)^2 + y^2 = 2}$$
$$\mathbf{y^2 + y^2 = 2}$$
$$\mathbf{2y^2 = 2}$$
$$\mathbf{y^2 = 1}$$
$$\mathbf{y = \sqrt{1}}$$
$$\mathbf{y = \pm 1}$$
$$\mathbf{at\ y = 1}$$
$$\mathbf{x = -1}$$
$$\mathbf{at\ y = -1}$$
$$\mathbf{x = 1.}$$

11) Given that $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 2 & x \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -\frac{3}{2} & -\frac{1}{2} \\ x & x - 12 \end{pmatrix}$, find x if **AB** has no inverse.

$$\mathbf{\begin{pmatrix} 0 & 1 \\ 2 & x \end{pmatrix} \text{ and } B = \begin{pmatrix} -\frac{3}{2} & -\frac{1}{2} \\ x & x - 12 \end{pmatrix}}$$

AB is singular;

$$\mathbf{AB = \begin{pmatrix} 0 & 1 \\ 2 & x \end{pmatrix}\begin{pmatrix} -\frac{3}{2} & -\frac{1}{2} \\ x & x - 12 \end{pmatrix}}$$
$$\mathbf{= \begin{pmatrix} x & x - 12 \\ -3 + x^2 & -1 + x^2 - 12x \end{pmatrix}}$$

For singular matrix;

$$\mathbf{x(-1 + x^2 - 12x) - (-3 + x^2)(x - 12) = 0}$$
$$\mathbf{(-x + x^3 - 12x^2) - (-3x + 36 + x^3 - 12x^2) = 0}$$
$$\mathbf{-x + x^3 - 12x^2 + 3x - 36 - x^3 + 12x^2 = 0}$$
$$\mathbf{-x + 3x - 36 = 0}$$
$$\mathbf{2x = 36}$$
$$\mathbf{x = 18.}$$

12) Given that $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$ and $\mathbf{C} = \mathbf{2AB} - \mathbf{A^2}$, determine the matrix $\mathbf{C}$.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$\mathbf{2AB} = 2\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

$$\mathbf{A^2} = \begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -8 & 9 \end{pmatrix}$$

$$\mathbf{C} = \mathbf{2AB} - \mathbf{A^2}$$

$$= \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ -8 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 0 \\ 8 & -3 \end{pmatrix}.$$

13) Matrix $\mathbf{A} = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 11 & 3 \\ 4 & 1 \end{pmatrix}$. Find C if BC = A.

$$\mathbf{A} = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 11 & 3 \\ 4 & 1 \end{pmatrix}$$

$$\text{Let } \mathbf{C} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\mathbf{BC} = \begin{pmatrix} 11 & 3 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} 11a + 3c & 11b + 3d \\ 4a + c & 4b + d \end{pmatrix}$$

But;

$$\mathbf{BC} = \mathbf{A}$$

$$\begin{pmatrix} 11a + 3c & 11b + 3d \\ 4a + c & 4b + d \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix}$$

$$11a + 3c = 2 \ldots\ldots\ldots \text{eq. } 1$$
$$4a + c = 3 \ldots\ldots\ldots\ldots \text{eq. } 2$$

$$11b + 3d = 4 \ldots\ldots\ldots\ldots \text{eq. } 3$$
$$4b + d = 6 \ldots\ldots\ldots\ldots \text{eq. } 4$$

Solving eq. 1 and eq. 2 simultaneously;

$$\begin{array}{r} 11a + 3c = 2 \\ \underline{12a + 3c = 9} \\ -a = -7 \end{array} \; -$$

$$a = 7$$

From eq. 2

$$28 + c = 3$$

$$c = 3 - 28$$

$$c = -25$$

Solving eq. 3 and 4 simultaneously;

$$\begin{array}{r} 11b + 3d = 4 \\ \underline{12b + 3d = 18} \\ -b = -14 \end{array} \; -$$

$$b = 14$$

From eq. 4

$$56 + d = 6$$

$$d = 6 = 56$$

$$d = -50$$

$$\mathbf{C} = \begin{pmatrix} 7 & 14 \\ -25 & -50 \end{pmatrix}.$$

14) Find the value of x for which the matrix $\mathbf{F} = \begin{pmatrix} x & -3 \\ 0 & x-1 \end{pmatrix}$ is singular.

$$\mathbf{F} = \begin{pmatrix} \mathbf{x} & \mathbf{-3} \\ \mathbf{0} & \mathbf{x-1} \end{pmatrix}$$

For singular matrix, Det = 0

$$\mathbf{x(x-1) - 0 = 0}$$

$$\mathbf{x(x-1) = 0}$$

$$\mathbf{x = 0}$$

or

$$\mathbf{x - 1 = 0}$$

$$\mathbf{x = 1.}$$

15) Given that $\mathbf{A} = \begin{pmatrix} 3 & -4 \\ 7 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & -1 & -2 \\ 4 & 2 & -3 \end{pmatrix}$, find the transpose of **AB**.

$$\begin{pmatrix} \mathbf{3} & \mathbf{-4} \\ \mathbf{7} & \mathbf{-1} \end{pmatrix} \textbf{ and } \mathbf{B} = \begin{pmatrix} \mathbf{0} & \mathbf{-1} & \mathbf{-2} \\ \mathbf{4} & \mathbf{2} & \mathbf{-3} \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} \mathbf{3} & \mathbf{-4} \\ \mathbf{7} & \mathbf{-1} \end{pmatrix}\begin{pmatrix} \mathbf{0} & \mathbf{-1} & \mathbf{-2} \\ \mathbf{4} & \mathbf{2} & \mathbf{-3} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{-16} & \mathbf{-11} & \mathbf{6} \\ \mathbf{-4} & \mathbf{-9} & \mathbf{-11} \end{pmatrix}$$

A is a 2 × 3 Matrix

$$\mathbf{(AB)^T = 3 \times 2 \text{ Matrix}}$$

$$\mathbf{AB} = \begin{pmatrix} \mathbf{-16} & \mathbf{-11} & \mathbf{6} \\ \mathbf{-4} & \mathbf{-9} & \mathbf{-11} \end{pmatrix}$$

$$\mathbf{(AB)^T} = \begin{pmatrix} \mathbf{-16} & \mathbf{-4} \\ \mathbf{-11} & \mathbf{-9} \\ \mathbf{6} & \mathbf{-11} \end{pmatrix}.$$

16) Given that $\mathbf{A} = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 4 & -1 \end{pmatrix}$, find the determinant of **AB**.

$$\mathbf{A} = \begin{pmatrix} \mathbf{4} & \mathbf{3} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} & \mathbf{-2} \end{pmatrix} \textbf{ and } \mathbf{B} = \begin{pmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{1} & \mathbf{-2} \\ \mathbf{4} & \mathbf{-1} \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} \mathbf{4} & \mathbf{3} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} & \mathbf{-2} \end{pmatrix}\begin{pmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{1} & \mathbf{-2} \\ \mathbf{4} & \mathbf{-1} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{15} & \mathbf{5} \\ \mathbf{-3} & \mathbf{6} \end{pmatrix}$$

$$\mathbf{Det = (15 \times 6) - (5 \times -3)}$$

$$\mathbf{= 90 + 15}$$

$$\mathbf{= 105.}$$

17) Use matrix method to solve the equation $\begin{matrix} 2x + 3y = 13 \\ 3x = 2y \end{matrix}$

$$\begin{pmatrix} 2 & 3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ 0 \end{pmatrix}$$

Det;

$$= -4 - 9$$

$$= -13$$

$$-\frac{1}{13}\begin{pmatrix} -2 & -3 \\ -3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{13} & \frac{3}{13} \\ \frac{3}{13} & -\frac{2}{13} \end{pmatrix}$$

$$\begin{pmatrix} \frac{2}{13} & \frac{3}{13} \\ \frac{3}{13} & -\frac{2}{13} \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{13} & \frac{3}{13} \\ \frac{3}{13} & -\frac{2}{13} \end{pmatrix}\begin{pmatrix} 13 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$x = 2$$

$$y = 3.$$

18) A matrix $\mathbf{P} = \begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Find the values of x and y if $\mathbf{PQ} = \mathbf{R}$ using matrix method.

$$\mathbf{P} = \begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } \mathbf{R} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\mathbf{PQ} = \begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

But;

$$\mathbf{PQ} = \mathbf{R}$$

$$\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\text{Det} = 6 - 4$$

$$= 2$$

$$\frac{1}{2}\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3.5 \\ 5 \end{pmatrix}$$

$$x = 3.5$$

$$y = 5.$$

19) Given that $\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$, find $\mathbf{A^{-1}}$ hence solve the equation $\begin{matrix} 3a + 2b = 12 \\ 4a - b = 5 \end{matrix}$

$$\mathbf{A = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}}$$

Det;

$$\mathbf{= -3 - 8}$$

$$\mathbf{= -11}$$

$$\mathbf{A^{-1} = -\frac{1}{11}\begin{pmatrix} -1 & -2 \\ -4 & 3 \end{pmatrix}}$$

$$\mathbf{= \begin{pmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{4}{11} & -\frac{3}{11} \end{pmatrix}}$$

From;

$$\mathbf{3a + 2b = 12}$$
$$\mathbf{4a - b = 5}$$

$$\mathbf{\begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}}$$

Solving it;

$$\mathbf{\begin{pmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{4}{11} & -\frac{3}{11} \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{4}{11} & -\frac{3}{11} \end{pmatrix}\begin{pmatrix} 12 \\ 5 \end{pmatrix}}$$

$$\mathbf{\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}}$$

$$\mathbf{a = 2}$$

$$\mathbf{b = 3.}$$

20) Given that $\mathbf{A} = \begin{pmatrix} 3 & 2 \\ -1 & 3 \end{pmatrix}$, find a matrix B if $AB = I$ hence determine the point where the lines $3x + 2y = 10$ and $3y - x = 4$ intersect.

$$\mathbf{A = \begin{pmatrix} 3 & 2 \\ -1 & 3 \end{pmatrix}}$$

Let B be $\mathbf{\begin{pmatrix} a & b \\ c & c \end{pmatrix}}$

$$\mathbf{AB = \begin{pmatrix} 3 & 2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}}$$

$$\mathbf{= \begin{pmatrix} 3a + 2c & 3b + 2d \\ -a + 3c & -b + 3d \end{pmatrix}}$$

But AB = I

$$\mathbf{\begin{pmatrix} 3a + 2c & 3b + 2d \\ -a + 3c & -b + 3d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$\mathbf{3a + 2c = 1 \ldots\ldots eq.1}$$

$$\mathbf{-a + 3c = 0}$$

$$\mathbf{-a = -3c}$$

$$\mathbf{a = 3c \ldots\ldots eq.2}$$

Substituting eq. 2 into eq. 1

$$\mathbf{9c + 2c = 1}$$

$$\mathbf{11c = 1}$$

$$\mathbf{c = \frac{1}{11}}$$

$$\mathbf{a = \frac{3}{11}}$$

$$\mathbf{3b + 2d = 0}$$

$$\mathbf{3b = -2d}$$

$$\mathbf{b = -\frac{2}{3}d \ldots\ldots eq.3}$$

$$\mathbf{-b + 3d = 1}$$

$$\mathbf{\frac{2}{3}d + 3d = 1 \ldots\ldots eq.4}$$

Substituting eq. 3 into eq. 4;

$$\mathbf{2d + 9d = 3}$$

$$\mathbf{11d = 3}$$

$$\mathbf{d = \frac{3}{11}}$$

$$\mathbf{b = -\frac{2}{3}\left(\frac{3}{11}\right)}$$

$$\mathbf{b = -\frac{2}{11}}$$

$$\mathbf{B = \begin{pmatrix} \frac{3}{11} & -\frac{2}{11} \\ \frac{1}{11} & \frac{3}{11} \end{pmatrix}}$$

B is the inverse of A

From;

$$\mathbf{3x + 2y = 10}$$

$$\mathbf{3y - x = 4}$$

We have;

$$\mathbf{\begin{pmatrix} 3 & 2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}}$$

$$\mathbf{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{11} & -\frac{2}{11} \\ \frac{1}{11} & \frac{3}{11} \end{pmatrix}\begin{pmatrix} 10 \\ 4 \end{pmatrix}}$$

$$\mathbf{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}}$$

$$\mathbf{x = 2}$$

$$\mathbf{y = 2.}$$

SECTION B.

21) (a) Given that matrix $\mathrm{M} = \begin{pmatrix} -3 & -2 \\ 4 & 3 \end{pmatrix}$, find M^2. (2 marks)

$$\mathbf{M} = \begin{pmatrix} -3 & -2 \\ 4 & 3 \end{pmatrix}$$

$$\mathbf{M}^2 = \begin{pmatrix} -3 & -2 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} -3 & -2 \\ 4 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

(b) In order to pay for a cinema ticket, a fixed price of Ksh. x is paid and Ksh. y for every movie watched. A ticket costs Ksh. 201 for 4 movies and Ksh. 245 for 5 movies.

i. Form two linear equations in *x* and *y* to represent the information above. (2 marks)

$$\mathbf{x + 4y = 201}$$
$$\mathbf{x + 5y = 245}$$

ii. Using a matrix method, determine the values of *x* and *y*. (4 marks)

$$\begin{pmatrix} 1 & 4 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 210 \\ 245 \end{pmatrix}$$

$$\mathbf{Det = 5 - 4}$$

$$= \mathbf{1}$$

Inverse;

$$= \begin{pmatrix} 5 & -4 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 5 & -4 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -4 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 210 \\ 245 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 25 \\ 44 \end{pmatrix}$$

$$\mathbf{x = ksh. 25}$$

$$\mathbf{y = ksh. 44.}$$

iii. Determine the price for watching 6 movies. (2 marks)

$$\mathbf{x + 6y = ?}$$

$$\mathbf{25 + 6(44)}$$

$$\mathbf{= ksh. 289.}$$

22) (a) Given that matrix $A = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$, find A^{-1}

$$A = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$$

$$\text{Det} = 9 - 8$$

$$= 1$$

$$A^{-1} = \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}.$$

(b) Two universities TEC and KCT purchased beans and rice. TEC bought 60 bags of beans and 80 bags of rice for a total of kshs. 240,000. KCT bought 150 bags of beans and 225 bags of rice for a total of shs. 637,500. Use the matrix method to find the price of one bag of each item. (5marks)

$$60b + 80r = 240,000$$
$$150b + 225r = 637,500$$

$$\begin{pmatrix} 60 & 80 \\ 150 & 225 \end{pmatrix}\begin{pmatrix} b \\ r \end{pmatrix} = \begin{pmatrix} 240,000 \\ 637,500 \end{pmatrix}$$

$$\text{Det} = 13,500 - 12,000$$

$$= 1,500$$

Inverse;

$$= \frac{1}{1500}\begin{pmatrix} 225 & -80 \\ -150 & 60 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{20} & -\frac{4}{75} \\ -\frac{1}{10} & \frac{1}{25} \end{pmatrix}$$

Solution is at;

$$\begin{pmatrix} \frac{3}{20} & -\frac{4}{75} \\ -\frac{1}{10} & \frac{1}{25} \end{pmatrix}\begin{pmatrix} 60 & 80 \\ 150 & 225 \end{pmatrix}\begin{pmatrix} b \\ r \end{pmatrix} = \begin{pmatrix} \frac{3}{20} & -\frac{4}{75} \\ -\frac{1}{10} & \frac{1}{25} \end{pmatrix}\begin{pmatrix} 240,000 \\ 637,500 \end{pmatrix}$$

$$\begin{pmatrix} b \\ r \end{pmatrix} = \begin{pmatrix} 2,000 \\ 1,500 \end{pmatrix}$$

Beans cos ksh. 2,000

Rice cost ksh. 1,500.

(c) The price of beans later decreased in the ratio 4: 5 while the price of rice increased by 20%. A businessman bought 20 bags of beans and 30 bags of rice. How much did he pay? (3 marks)

$$\text{Beans} = \frac{4}{5} \times 2000$$

$$= \text{ksh. } 1,600$$

$$\text{Rice} = \frac{120}{150} \times 1500$$

$$= \text{kshs. } 1,800$$

$$20(1600) + 30(1800)$$

$$= \text{ksh. } 86,000.$$

23) (a) Given the matrix , $A = \begin{pmatrix} 4 & -1 \\ 15 & -4 \end{pmatrix}$, find A^2

$$\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 15 & -4 \end{pmatrix}$$

$$\mathbf{A^2} = \begin{pmatrix} 4 & -1 \\ 15 & -4 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 15 & -4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

(b) Two businessmen Thomas and Stephen bought radios and juice blenders at sh x per radio and sh y per juice blender. Thomas paid a total of shs. 60 000 for 15 radios and 20 juice blenders while Stephen paid a total of shs. 64 000 for 14 radios and 24 juice blenders.

i. Form a matrix equation to represent this information (2 marks)

$$\mathbf{15x + 20y = 60,000}$$

$$\mathbf{14x + 24y = 64,000.}$$

ii. Use the matrix method to find the cost of one radio and one juice blender. (4 marks)

$$\begin{pmatrix} 15 & 20 \\ 14 & 24 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 60,000 \\ 64,000 \end{pmatrix}$$

Det;

$$= 360 - 280$$

$$= 80$$

Inverse;

$$= \frac{1}{80}\begin{pmatrix} 24 & -20 \\ -14 & 15 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{10} & -\frac{1}{4} \\ -\frac{7}{40} & \frac{3}{16} \end{pmatrix}$$

Solution will be at;

$$\begin{pmatrix} \frac{3}{10} & -\frac{1}{4} \\ -\frac{7}{40} & \frac{3}{16} \end{pmatrix}\begin{pmatrix} 15 & 20 \\ 14 & 24 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{10} & -\frac{1}{4} \\ -\frac{7}{40} & \frac{3}{16} \end{pmatrix}\begin{pmatrix} 60,000 \\ 64,000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2000 \\ 1500 \end{pmatrix}$$

Radio cost ksh. 2,000

Juice blender cost ksh. 1,500.

(c) Thomas sold all his radios and juice blenders at a profit of 20% per radio and 25% per juice blender. Stephen sold all his radios and juice blenders at a profit of 25% per radio and 20% per juice blender. Calculate the profit that each businessman made. (2 marks)

Thomas;

$$15x + 20y = 60,000$$

$$\frac{120}{100} \times 2000$$

$$= ksh. 2,400$$

$$\frac{125}{100} \times 1,500$$

$$= ksh. 1,875$$

Total S. P;

$$= 15(2,400) + 20(1,875)$$

$$= ksh. 73,500$$

Thomas profit

$$= 73,500 - 60,000$$

$$= kshs, 13,500.$$

Stephen;

$$14x + 24y = 64,000$$

$$\frac{125}{100} \times 2000$$

$$= ksh. 2,500$$

$$\frac{120}{100} \times 1,500$$

$$= ksh. 1,800$$

Total S. P;

$$= 14(2500) + 24(1800)$$

$$= ksh. 78,200$$

Stephen Profit;

$$= 78,200 - 64,000$$

$$= ksh. 14,200.$$

24) (a) Given the matrix $T = \begin{pmatrix} 3 & 4 \\ 7 & 12 \end{pmatrix}$, find T^{-1}

$$\mathbf{T = \begin{pmatrix} 3 & 4 \\ 7 & 12 \end{pmatrix}}$$

$$\mathbf{Det = 36 - 28}$$

$$\mathbf{= 8}$$

$$\mathbf{T^{-1} = \frac{1}{8}\begin{pmatrix} 12 & -4 \\ -7 & 3 \end{pmatrix}}$$

$$\mathbf{= \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{7}{8} & \frac{3}{8} \end{pmatrix}}$$

(b) Dorothy, a business lady, bought 3 blow dryers and 4 shaving machines at a cost of $sh.$ 12,000. Had she bought 7 blow dryers and 12 shaving machines, she would have spent $sh.$ 20,000 more.

i. Taking x and y to be the prices of a blow dryer and a shaving machine respectively, form a matrix equation to represent this information. (2 marks)

$$\mathbf{3x + 4y = 12,000}$$

$$\mathbf{7x + 12y = 20,000.}$$

ii. Use the matrix method to find the cost of one blow dryer and one shaving machine. (4marks)

$$\mathbf{\begin{pmatrix} 3 & 4 \\ 7 & 12 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12,000 \\ 20,000 \end{pmatrix}}$$

$$\mathbf{Det = 36 - 28}$$

$$\mathbf{= 8}$$

$$\mathbf{Inverse = \frac{1}{8}\begin{pmatrix} 12 & -4 \\ -7 & 3 \end{pmatrix}}$$

$$\mathbf{= \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{7}{8} & \frac{3}{8} \end{pmatrix}}$$

The solution is at;

$$\mathbf{\begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{7}{8} & \frac{3}{8} \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 7 & 12 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{7}{8} & \frac{3}{8} \end{pmatrix}\begin{pmatrix} 12,000 \\ 20,000 \end{pmatrix}}$$

$$\mathbf{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2,000 \\ 1,500 \end{pmatrix}}$$

Blow dryer cost ksh. 2,000

Shaving machine cost Ksh. 1,500.

iii. Dorothy discovered that in another shop the price of a blow dryer is 20% less while that of shaving machine is 10% higher. Calculate how much she would have saved if she bought them from that shop. (2 marks)

$$\mathbf{\frac{80}{100} \times 2000}$$

$$\mathbf{= ksh. 1,600}$$

$$\mathbf{\frac{110}{100} \times 1,500}$$

$$\mathbf{= ksh. 1,650}$$

New buying Price;

$$\mathbf{3(1,600) + 4(1,650)}$$

$$\mathbf{= ksh. 11,400}$$

Amount saved;

$$\mathbf{= 12,000 - 11,400}$$

$$\mathbf{= ksh. 600.}$$

25) (a) Express A^{-1} in the form $\begin{pmatrix} a & c \\ c & d \end{pmatrix}$ given that $A = \begin{pmatrix} 1 & 1 \\ 8 & 5 \end{pmatrix}$. (2 marks)

$$A = \begin{pmatrix} 1 & 1 \\ 8 & 5 \end{pmatrix}$$

Det;

$$= 5 - 8$$

$$= -3$$

$$A^{-1} = -\frac{1}{3}\begin{pmatrix} 5 & -1 \\ -8 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{5}{3} & \frac{1}{3} \\ \frac{8}{3} & -\frac{1}{3} \end{pmatrix}.$$

(b) A shirt and a blouse cost a total of Ksh. 2,700. When the price of the shirt is increased by 20% and that of the blouse is decreased by 25%, their total cost is Ksh. 2,565. By letting the price of the shirt to be x and that of the blouse to be y;

i. Form two simplified equations in x and y to represent the above information. (3 marks)

$$x + y = 2,700 \ldots\ldots \text{eq. } 1$$

$$\frac{120}{100}x + \frac{75}{100}y = 2,565$$

Multiplying through by 100;

$$120x + 75y = 256,500$$

Dividing through by 15;

$$8x + 5y = 17,100 \ldots . \text{eq. } 2.$$

ii. Using matrix method, find the cost of each item. (5 marks)

$$\begin{pmatrix} 1 & 1 \\ 8 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2,700 \\ 17,100 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 \\ 8 & 5 \end{pmatrix}$$

Det;

$$= 5 - 8$$

$$= -3$$

Inverse;

$$= -\frac{1}{3}\begin{pmatrix} 5 & -1 \\ -8 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{5}{3} & \frac{1}{3} \\ \frac{8}{3} & -\frac{1}{3} \end{pmatrix}$$

The solution is at;

$$\begin{pmatrix} -\frac{5}{3} & \frac{1}{3} \\ \frac{8}{3} & -\frac{1}{3} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 8 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{5}{3} & \frac{1}{3} \\ \frac{8}{3} & -\frac{1}{3} \end{pmatrix}\begin{pmatrix} 2,700 \\ 17,100 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1,200 \\ 1,500 \end{pmatrix}.$$

26) (a) Given that $A = \begin{pmatrix} 8 & 5 \\ 6 & 4 \end{pmatrix}$, find the inverse of A. (2 marks)

$$A = \begin{pmatrix} 8 & 5 \\ 6 & 4 \end{pmatrix}$$

Det;

$$= 32 - 30$$

$$= 2$$

$$A^{-1} = \frac{1}{2}\begin{pmatrix} 4 & -5 \\ -6 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -2.5 \\ -3 & 4 \end{pmatrix}.$$

(b) A student bought 16 exercise books and 10 pens at a total cost of Ksh. 1 018. If she had bought 12 exercise books and 8 pens, she would have spent Ksh 242 less.

i. Form a matrix equation to represent the information above. (2 marks)

$$16b + 10p = 1018$$

$$12b + 8p = 776$$

Simmplified form;

$$8b + 5p = 509$$

$$6b + 4p = 338.$$

ii. Using the inverse of A in (a) above, determine the price of each item. (4 marks)

$$A^{-1} = \begin{pmatrix} 2 & -2.5 \\ -3 & 4 \end{pmatrix}$$

From;

$$\begin{pmatrix} 8 & 5 \\ 6 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 509 \\ 338 \end{pmatrix}$$

Solution will be;

$$\begin{pmatrix} 2 & -2.5 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 8 & 5 \\ 6 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -2.5 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 509 \\ 338 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 48 \\ 25 \end{pmatrix}$$

A book cost ksh. 48

A pen cost ksh. 25.

(c) Find the total cost of 4 books and 5 pens by using matrices of orders 1×2 and 2×1 respectively. (2 marks)

$$4b + 5p$$

Using 1×2 Matrix;

$$\begin{pmatrix} 4 & 5 \end{pmatrix}\begin{pmatrix} 48 \\ 25 \end{pmatrix}$$

$$= \text{ksh. } 317$$

Using a 2×1 Matrix;

$$\begin{pmatrix} 4 \\ 5 \end{pmatrix}\begin{pmatrix} 48 & 25 \end{pmatrix}$$

Incompatible Matrix.

27) To watch a movie, a fixed amount of Sh. x is charged and a further Sh. y for every movie watched. It cost Sh. 80 to watch 2 movies and Sh. 125 to watch 5 movies.

(a) Form two equations in *x* and *y* from the information above. (2 marks)

$$\mathbf{x + 2y = 80}$$

$$\mathbf{x + 5y = 125.}$$

(b) Express the equations in (a) above in matrix form. (1 mark)

$$\begin{pmatrix} 1 & 2 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 80 \\ 125 \end{pmatrix}$$

(c) Use a matrix method to find the values of *x* and *y* and hence state the standing charge for watching the movie. (5 marks)

From;

$$\begin{pmatrix} 1 & 2 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 80 \\ 125 \end{pmatrix}$$

Det;

$$= 5 - 2$$

$$= 3$$

Inverse;

$$= \frac{1}{3}\begin{pmatrix} 5 & -2 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{5}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

The solution will be;

$$\begin{pmatrix} \frac{5}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{5}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} 80 \\ 125 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 50 \\ 15 \end{pmatrix}$$

$$\mathbf{x = ksh. 50}$$

$$\mathbf{y = ksh. 15}$$

Standing charge;

$$\mathbf{= kshs. 50.}$$

(d) Find the cost of watching 8 movies. (2 marks)

$$= x + 8y$$

$$= 50 + (15 \times 8)$$

$$= ksh. 170.$$

28) (a) Given that $T^{-1} = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{7}{8} & \frac{3}{8} \end{pmatrix}$, find the matrix **T** (2 marks)

$$TT^{-1} = I$$

$$\text{Let } T = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$TT^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{7}{8} & \frac{3}{8} \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{7}{8} & \frac{3}{8} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{2}a - \frac{7}{8}b & -\frac{1}{2}a + \frac{3}{8}b \\ \frac{3}{2}c - \frac{7}{8}d & -\frac{1}{2}c + \frac{3}{8}d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{3}{2}a - \frac{7}{8}b = 1 \ldots\ldots\ldots..\text{eq. }1$$

$$-\frac{1}{2}a + \frac{3}{8}b = 0$$

$$-\frac{1}{2}a = -\frac{3}{8}b$$

$$a = \frac{3}{4}b \ldots\ldots\ldots\text{eq. }2$$

Substituting in eq. 1, we have;

$$\frac{3}{2}\left(\frac{3}{4}b\right) - \frac{7}{8}b = 1$$

$$\frac{9}{8}b - \frac{7}{8}b = 1$$

$$\frac{1}{4}b = 1$$

$$b = 4$$

$$a = \frac{3}{4}(4)$$

$$a = 3$$

$$\frac{3}{2}c - \frac{7}{8}d = 0$$

$$\frac{3}{2}c = \frac{7}{8}d$$

$$c = \frac{7}{12}d \ldots..\text{eq. }3$$

$$-\frac{1}{2}c + \frac{3}{8}d = 1 \ldots\ldots\text{eq. }4$$

Substituting eq. 3 into eq. 4;

$$-\frac{1}{2}\left(\frac{7}{12}d\right) + \frac{3}{8}d = 1$$

$$-\frac{7}{24}d + \frac{3}{8}d = 1$$

$$\frac{1}{12}d = 1$$

$$d = 12$$

$$c = \frac{7}{12}(12)$$

$$c = 7$$

$$T = \begin{pmatrix} 3 & 4 \\ 7 & 12 \end{pmatrix}.$$

(b) Two ladies bought trouser suits and skirt suits from a boutique at Ksh x and sh y per item respectively. Jane paid Ksh 12,000 for 3 trouser suits and 4 skirt suits. Lucy paid Ksh 32,000 for 7 trouser suits and 12 skirt suits.

i. Form a matrix equation to represent this information (2 marks)

$$3x + 4y = 12{,}000$$

$$7x + 12y = 32{,}000$$

Matrix equation are;

$$\begin{pmatrix} 3 & 4 \\ 7 & 12 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12{,}000 \\ 32{,}000 \end{pmatrix}.$$

ii. Use matrix method to find the cost of one trouser suit and one skirt suit. (4 marks)

From;

$$\begin{pmatrix} 3 & 4 \\ 7 & 12 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12,000 \\ 32,000 \end{pmatrix}$$

Inverse;

$$= \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{7}{8} & \frac{3}{8} \end{pmatrix}$$

Solution will be;

$$\begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{7}{8} & \frac{3}{8} \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 7 & 12 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{7}{8} & \frac{3}{8} \end{pmatrix}\begin{pmatrix} 12,000 \\ 32,000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2,000 \\ 1,500 \end{pmatrix}$$

Trouser suit = ksh. 2, 000

Skirt suit = Ksh. 1, 500.

(c) By using 1×2 and 2×1 matrices respectively, calculate the total cost of 5 trouser suits and 7 skirt suits. (2 marks)

Using 1×2 Matrix

$$\begin{pmatrix} 5 & 7 \end{pmatrix}\begin{pmatrix} 2,000 \\ 1,500 \end{pmatrix}$$

= ksh. 20, 500

Using 2×1 matrix;

$$\begin{pmatrix} 5 \\ 7 \end{pmatrix}\begin{pmatrix} 2,000 & 1,500 \end{pmatrix}$$

The matrix is incompatible.

29) In a week, a school buys 15 bags of maize, 8 bags of beans, 16 bags of rice and 4 bags of sugar. The prices per bag of each commodity are sh.1400, sh.2600, sh.1600 and sh. 1500 respectively.

(a) Form 1×4 matrix to represent the number of bags bought by the school. (1 mark)

$$\begin{pmatrix} \mathbf{15} & \mathbf{8} & \mathbf{16} & \mathbf{4} \end{pmatrix}$$

(b) Form a 4×1 matrix representing the cost of the above commodities. *(*1 mark)

$$\begin{pmatrix} \mathbf{1,400} \\ \mathbf{2,600} \\ \mathbf{1,600} \\ \mathbf{1,500} \end{pmatrix}$$

(c) Find by matrix method the amount of money the school spends per week. (2 marks)

$$\begin{pmatrix} \mathbf{15} & \mathbf{8} & \mathbf{16} & \mathbf{4} \end{pmatrix}\begin{pmatrix} \mathbf{1,400} \\ \mathbf{2,600} \\ \mathbf{1,600} \\ \mathbf{1,500} \end{pmatrix}$$

$$= \mathbf{21,000 + 20,800 + 25,600 + 6,000}$$

$$= \mathbf{ksh.\ 73,400.}$$

(d) A transporter has two types of buses; type A and type B. Type A has a capacity for 52 passengers and 200kg of luggage while type B has a capacity for 32 passengers and 300kg of luggage. At full capacity, the revenue from type A is sh. 62,000 and from type B is sh. 47,000. Taking sh. x to be the fare per passenger and sh. y to be the charge per kilogram of luggage;

i. Form a pair of linear equations to represent this information. (1 mark)

$$\mathbf{52x + 200y = 62,000}$$

$$\mathbf{32x + 300y = 47,000.}$$

ii. Use matrix method to find the values of *x* and *y*. (5 marks)

From;

$$\mathbf{52x + 200y = 62,000}$$

$$\mathbf{32x + 300y = 47,000.}$$

$$\begin{pmatrix} \mathbf{52} & \mathbf{200} \\ \mathbf{32} & \mathbf{300} \end{pmatrix}\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} \mathbf{62,000} \\ \mathbf{47,000} \end{pmatrix}$$

Det;

$$= \mathbf{(300 \times 52) - (32 \times 200)}$$

$$= \mathbf{9,200}$$

Inverse;

$$= \frac{\mathbf{1}}{\mathbf{9,200}}\begin{pmatrix} \mathbf{300} & \mathbf{-200} \\ \mathbf{-32} & \mathbf{52} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\mathbf{3}}{\mathbf{92}} & -\frac{\mathbf{1}}{\mathbf{46}} \\ -\frac{\mathbf{2}}{\mathbf{575}} & \frac{\mathbf{13}}{\mathbf{2300}} \end{pmatrix}$$

Solution will be;

$$\begin{pmatrix} \frac{\mathbf{3}}{\mathbf{92}} & -\frac{\mathbf{1}}{\mathbf{46}} \\ -\frac{\mathbf{2}}{\mathbf{575}} & \frac{\mathbf{13}}{\mathbf{2300}} \end{pmatrix}\begin{pmatrix} \mathbf{52} & \mathbf{200} \\ \mathbf{32} & \mathbf{300} \end{pmatrix}\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\mathbf{3}}{\mathbf{92}} & -\frac{\mathbf{1}}{\mathbf{46}} \\ -\frac{\mathbf{2}}{\mathbf{575}} & \frac{\mathbf{13}}{\mathbf{2300}} \end{pmatrix}\begin{pmatrix} \mathbf{62,000} \\ \mathbf{47,000} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} \mathbf{10,00} \\ \mathbf{50} \end{pmatrix}$$

$$\mathbf{x = ksh.\ 1,000}$$

$$\mathbf{y = ksh.\ 50.}$$

30) (a) Given that $P = \begin{pmatrix} 3 & -1 \\ x & 5 \end{pmatrix}$ and $Q = \begin{pmatrix} x & -11 \\ 8 & 18 \end{pmatrix}$, find x if $|P^2 - Q| = 1$

$$P = \begin{pmatrix} 3 & -1 \\ x & 5 \end{pmatrix} \text{ and } Q = \begin{pmatrix} x & -11 \\ 8 & 18 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 3 & -1 \\ x & 5 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ x & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 9 - x & -8 \\ 8x & -x + 25 \end{pmatrix}$$

$$P^2 - Q = \begin{pmatrix} 9 - x & -8 \\ 8x & -x + 25 \end{pmatrix} - \begin{pmatrix} x & -11 \\ 8 & 18 \end{pmatrix}$$

$$= \begin{pmatrix} 9 - 2x & 3 \\ 8x - 8 & -x + 7 \end{pmatrix}$$

But;

$$|P^2 - Q| = 1$$

$$\text{Det} = 1;$$

$$[(9 - 2x)(7 - x)] - [(3)(8x - 8)] = 1$$

$$[63 - 9x - 14x + 2x^2] - [24x - 24] = 1$$

$$[63 - 23x + 2x^2] - [24x - 24] = 1$$

$$63 - 23x + 2x^2 - 24x + 24 = 1$$

$$2x^2 - 47x + 86 = 0$$

Using quadrtaic formula;

$$x = \frac{47 \pm \sqrt{2209 - 688}}{4}$$

$$= \frac{47 \pm 39}{4}$$

$$x = 21.5$$

or

$$x = 2.$$

(b) John intends to buy 10 exercise books, 3 fountain pens and a bottle of ink while Ken intends to buy 12 exercise books, 2 fountain pens and 2 bottles of ink. The items can be bought from two stationers; A and B. Stationer A sells an exercise book at sh. 45, a fountain pen at sh. 100 and a bottle of ink at sh. 80. Stationer B sells an exercise book at sh. 50, a fountain pen at sh. 95 and a bottle of ink at sh. 75.

i. Use a 2×3 matrix to represent number of items bought by John and Ken. (1 mark)

$$\begin{pmatrix} 10 & 3 & 1 \\ 12 & 2 & 2 \end{pmatrix}$$

ii. Use a 3×2 matrix to represent the prices of the items from the two stationers. (1 mark)

$$\begin{pmatrix} 45 & 50 \\ 100 & 95 \\ 80 & 75 \end{pmatrix}$$

iii. Using the matrices in (a) and (b) above, determine the total cost that would be incurred by John and Ken when they buy from the two stationers. (3 marks)

$$\begin{pmatrix} 10 & 3 & 1 \\ 12 & 2 & 2 \end{pmatrix}\begin{pmatrix} 45 & 50 \\ 100 & 95 \\ 80 & 75 \end{pmatrix}$$

$$= \begin{pmatrix} (10 \times 45) + (3 \times 100) + (1 \times 80) & (10 \times 50) + (3 \times 95) + (1 \times 75) \\ (12 \times 45) + (2 \times 100) + (1 \times 80) & (12 \times 50) + (2 \times 95) + (1 \times 75) \end{pmatrix}$$

$$= \begin{pmatrix} 830 & 860 \\ 900 & 940 \end{pmatrix}$$