Chapter Ten

ALGEBRAIC EXPRESSIONS

10.1: Symbolic Representation

Consider the following:

(i) Wakio had 5 goats and Baraza 19 goats. How many goats did they have altogether?

If instead Wakio had P goats and Baraza Q goats, how many goats would they have altogether?

(ii) Miriam buys a number of oranges from Mponda's kiosk. She then moves to Mary's kiosk and buys twice as many oranges as she had from the first kiosk. Using a letter of your choice, write down an expression for the total number of oranges bought.

If we choose n to represent the number of oranges bought at Mponda's kiosk, then the number bought from the Mary's kiosk would be 2n. Therefore, the expression for total number of oranges would be (n + 2n).

In algebra, (i) letters are used to represent numbers.

(ii) 2n means 2 x n.

Exercise 10.1

1. Use letters to represent the following statements:
   (a) The sum of two consecutive integers.
   (b) A number is multiplied by 3, then six is added to the result.
   (c) The area of a rectangle whose length is $2\frac{1}{2}$ times its width.
   (d) The distance covered in a given time at a given speed.
   (e) The number of days in m weeks.
   (f) The number of hours in $\frac{n}{t}$ days.
2. John is twice as old as his brother Kogo, and their sister Jane is 7 years younger than Kogo. Write down an expression for the sum of their ages.
3. Three children are given a certain number of bananas by their mother. If they share the bananas equally, how many does each get?
4. Wafula earns twice as much as his wife Grace. Write down an expression for the difference between their earnings.

5. A number is added to $\frac{2}{3}$ of itself. If the sum is doubled, write down the final expression.

6. For the figure below, express in terms of a, b, c and d:
   (a) the area of each of the rectangles, A, B, C and D.
   (b) the perimeter of each of the rectangles A, B, C and D.
   (c) the total area of the four rectangles A, B, C and D.

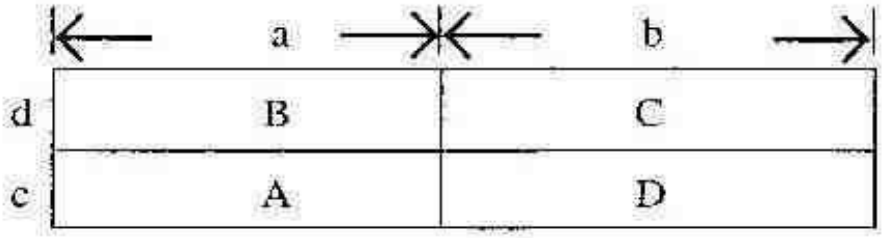

Fig. 10.1

7. George is ten years older than his brother Sam. Find an expression for:
   (a) the sum of their ages.
   (b) the sum of their ages in 5 years time.
   (c) the product of their ages three years ago.

8. In a mathematics test, six pupils obtained the following scores; 40, c, 20, d, e and f. Find their average score.

9. A man spends half of his monthly salary on food and one eighth on water and electricity. Write down and expression for the remainder of his salary.

10. In the figure below, ABCD is a square. DEFC, BCGH and AEFB are all rectangles. Find:
    (a) CF in terms of a and b.
    (b) the area of rectangle CDEF.

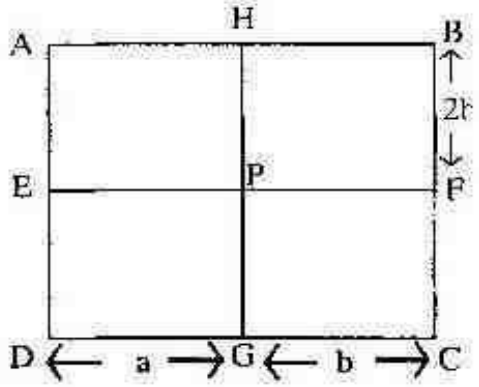

Fig. 10.2

11. An expression is obtained from three numbers by subtracting the difference between the first two from the third and adding twice the result to the first number. Write down the expression.
12. A man whose stride is $\frac{3}{4}$ m long walks all the way round a rectangular plot measuring 'a' m by 'b' m. Find an expression for the total number of steps.

10.2: Simplification of Algebraic Expressions

Like and Unlike Terms

Whereas we may add 3 cows to 4 cows to get 7 cows, the result of adding 3 cows to 4 houses cannot be expressed in any simpler form.

Similarly, $3a + 4a = 7a$, but $3a + 4b$ cannot be simplified further. 3a and 4a are like terms, while 3a and 4b are unlike terms.

Example 1

Simplify: $3x + 4y - x + z + 3y$

Solution

$$\begin{aligned} 3x + 4y - x + z + 3y &= (3x - x) + (4y + 3y) + z \\ &= 2x + 7y + z \end{aligned}$$

Example 2

Simplify: $2x - 6y - 4x + 5z - y$

Solution

$$\begin{aligned} 2x - 6y - 4x + 5z - y &= 2x - 4x - 6y - y + 5z \\ &= (2x - 4x) - (6y + y) + 5z \\ &= -2x - 7y + 5z \end{aligned}$$

Note:

$-6y - y = -(6y + y)$

Example 3

Simplify: $\frac{1}{2}a - \frac{1}{3}b + \frac{1}{4}a$

Solution

The L.C.M. of 2, 3 and 4 is 12.

Therefore $\frac{1}{2}a - \frac{1}{3}b + \frac{1}{4}a = \frac{6a-4b+3a}{12}$

$= \frac{6a+3a-4b}{12}$

$= \frac{9a-4b}{12}$

Example 4

Simplify: $\frac{a+b}{2} - \frac{2a-b}{3}$

Solution

$\frac{a+b}{2} - \frac{2a-b}{3} = \frac{3(a+b)-2(2a-b)}{6}$

$= \frac{3a+3b-4a+2b}{6}$

$= \frac{3a-4a+3b+2b}{6}$

$= \frac{-a+5b}{6}$

Exercise 10.2

Simplify where possible:

1. (a) $2a + 3b + 4a - b$ (b) $-10k + 2m - 3k - 5m$
2. (a) $7t + 2p + 3t + 5p$ (b) $6x - 9x - 2y + 9y$
3. (a) $2a + 2b + 2c$ (b) $0.5r + 0.85 - 0.1r$
4. (a) $2a + 3b + 4a - b$ (b) $0.8r + 0.26 - 0.6r + 7c - 0.2r$
5. (a) $-4z - 7d + 2z$ (b) $8x - 2y - 2\frac{1}{2}x - 3y$
6. (a) $8w - p - 2w + 11p$ (b) $\frac{1}{3}r - \frac{1}{4}s + \frac{1}{6}s$
7. (a) $\frac{2a}{3} - \frac{1}{8}b + \frac{1}{6}a$ (b) $-\frac{t}{9} + \frac{5k}{6} - \frac{t}{3}$
8. (a) $\frac{2x}{3} + \frac{2y}{5} - \frac{4x}{3} - \frac{y}{5}$ (b) $\frac{3d}{4} - \frac{p}{8} - \frac{d}{2} + \frac{p}{4}$
9. (a) $\frac{1}{10} + \frac{d}{10} - d + 7a$ (b) $\frac{3x}{2} - \frac{5y}{6} + \frac{y}{4}$

10. In each of the following, pick the term which is not like the others:
    (a) $3a, 5a, 7a, 2a^2, 10a$
    (b) $m^5, 5m^3, 6m^5, 3m^5, 11m^5$
    (c) $a^2b, ab, ba^2, 5a^2b, 6ba^2$
    (d) $2cd, dc, -cd, c^2d, -\frac{3}{4}cd$

In general, terms are called 'like terms' if they have the same letters raised to the same power. Otherwise they are unlike terms. For example, $3a$ and $3a^2$ are unlike terms, while a^2 and $3a^2$ are like terms.

Example 5

Simplify: $5a + 2a^2 + 10a$

Solution

$$\begin{aligned} 5a + 2a^2 + 10a &= 5a + 10a + 2a^2 \\ &= 15a + 2a^2 \end{aligned}$$

Example 6

Simplify: $a^2b - 2b^3c + 3a^2b + b^3c$

Solution

$$\begin{aligned} a^2b - 2b^3c + 3a^2b + b^3c &= a^2b + 3a^2b - 2b^3c + b^3c \\ &= 4a^2b - b^3c \end{aligned}$$

Note:

W and w are unlike terms.

Exercise 10.3

Simplify where possible:

1. (a) $p^2 - 3p^2 + 7p^2 + 8p^2$ (b) $y^3 + 3y + 2y^3 + y$
2. (a) $x^2 + x - 2x^2 + 5x$ (b) $3W^5 + 3w^5 + 6w^5 - 4w$
3. (a) $a^2b + 2ba + 3ba^2 - ab$ (b) $a^2b + b^2a + 3ab^2$
4. (a) $pqr + rpq + qrp + 2pq$ (b) $u^3vw + uv^3w + uvw^3$
5. (a) $4R^2 - 2r^2 + 5R^4 + r^2$ (b) $\frac{1}{4}p^2q + \frac{1}{2}qp^2 - \frac{1}{2}q^2p$
6. (a) $\frac{1}{3}a^3 + \frac{2}{5}a^3 - \frac{1}{6}a^3$ (b) $0.5st + \frac{3}{4}t^2s + \frac{1}{10}ts - 5$

10.3: Brackets

In algebra, brackets serve the same purpose as they do in arithmetic.

Example 7

Remove the brackets and simplify:

(a) $3(a + b) - 2(a - b)$ (b) $5 - (6a - 3b - 8)$

(c) $\frac{1}{3}a + 3(5a + b - c)$ (d) $\frac{2}{y}(\frac{1}{2}y + y^2 + y^3)$

(e) $2b + 3\{3 - 2(a - 5)\}$

Solution

(a) $3(a + b) - 2(a - b) = 3a + 3b - 2a + 2b$
$= 3a - 2a + 3b + 2b$
$= a + 5b$

(b) $5 - 6(a - 3b - 8) = 5 - 6a + 18b + 48$
$= 53 - 6a + 18b$

(c) $\frac{1}{3}a + 3(5a + b - c) = \frac{1}{3}a + 15a + 3b - 3c$
$= 15\frac{1}{3}a + 3b - 3c$

(d) $\frac{2}{y}(\frac{1}{2}y + y^2 + y^3) = \frac{2}{y} \times \frac{1}{2}y + \frac{2}{y} \times y^2 + \frac{2}{y} \times y^3$
$= 1 + 2y + 2y^2$

(e) $2b + a\{3 - 2(a - 5)\} = 2b + a\{3 - 2a + 10\}$
$= 2b + 3a - 2a^2 + 10a$
$= 2b + 3a + 10a - 2a^2$
$= 2b + 13a - 2a^2$

Exercise 10.4

Remove the brackets and simplify:

1. (a) $3(r + s) + 4(r + s)$ (b) $4(2a + 3) - 3(5a - 6)$
2. (a) $(3 - y) - 2(x - y + 2)$ (b) $(3 - y) - 3(y - x - 2)$
3. (a) $2p(q + r) - r(p + q)$ (b) $\frac{4(x+1)-3(x-1)}{6}$

4. (a) $\frac{3}{4}y(5y+3)-3y(y+7)$ (b) $\frac{1}{2}w(8w-2m)+\frac{1}{3}(m-w^2)$

5. (a) $-\frac{1}{2}xy(x-xy)-x(xy-x^2)$ (b) $\frac{(x-2x+3)}{x}$

6. (a) $\frac{1}{y}(2y^2+3ay)$ (b) $\frac{4}{am}(a^2m^3+a)+\frac{1}{2}(a^2+m^3)$

7. (a) $2\{x+3(x+2y)\}$ (b) $a\{3(b+c)+4(c+a)\}$

8. (a) $\frac{1}{2}\{x-4(2y-3x)\}$ (b) $\frac{1}{4}\{x-(2y-3x)\}$

9. (a) $3\{2p-\frac{1}{3}(p-q+r)\}$ (b) $3y-4\{3-(y+2)\}$

10. (a) $-\{(m-2p-5)-3(2m+4p-3)\}$

(b) $\frac{1}{y}\{3(3T-ST)-(4ST-6T)\}$

You have learnt to remove brackets in some expressions like $3(m+n) = 3m + 3n$. This process is called **expansion.**

Sometimes the reverse process is required, e.g., in $3m + 3n$, since 3 is a common factor.

Therefore, $3m + 3n = 3(m + n)$.

Copy and complete the following:

(i) $2a + 2b = 2(\_\_\_\_ + \_\_\_\_)$

(ii) $4c - 4d = \_\_\_\_ (c - d)$

(iii) $3a + 6b = \_\_\_\_ (\_\_\_\_ + \_\_\_\_)$

(iv) $3a + 6ab - 9a^2b = \_\_\_\_ (1 + 2b - 3ab)$

What you have done in (i) – (iv) above is called **factorisation.**

Example 8

Factorise each of the following:

(a) $2a + 4b + 3a + 6b$ (b) $ar^3 + ar^4 + ar^5$

(c) $a^2p^3 - ap^2 + a^3p$ (d) $4x^2y + 20x^4y^3 - 36x^3y$

Solution

(a) $2a + 4b + 3a + 6b = 2a + 3a + 4b + 6b$

$= 5a + 10b$

5 is common to both

$\therefore 5a + 10b = 5(a + 2b)$

(b) $ar^3 + ar^4 + ar^5$

ar^3 is common

$\therefore r^3 + ar^4 + ar^5 = ar^3(1 + r + r^2)$

(c) $a^3p^3 - ap^2 + a^3p$

ap is common

$\therefore a^3p^3 - ap^2 + a^3p = ap(ap^2 - p + a^2)$

(d) $4x^2y + 20x^4y^2 - 36x^3y$

$4x^2y$ is common,

$\therefore 4x^2y + 20x^4y^2 - 36x^3y = 4x^2y(1 + 5x^2y - 9x)$

Exercise 10.5

Factorise each of the following:

1. (a) $3p + 3r + 3q + 3r$ (b) $4k + 6r + 2s + 2r$
2. (a) $5a - 10b + 5$ (b) $28 - 21w + 14t$
3. (a) $6a + 18b + 27c - 12d$ (b) $8x + 16y - 32k - 64p$
4. (a) $cd^2 - c^2d$ (b) $a^3b + a^2b^2 - ab^3$
5. (a) $6a + 18a^2b - 27ac + 12a^3d$ (b) $4a^2b + 24a^2c - 14a^3d$
6. (a) $4a^3b + 6a^2b - 9ab^2$ (b) $4pqr^2 + 6p^2qr^2 - 2pq^2r^2$
7. (a) $-27p^2q^2 + 6p^3q^5 - 3p^4q^3$ (b) $x^4y^6 + x^3y^5 + 3x^3y^4$
8. (a) $p^3q^2 + p^3q^3$ (b) $28x^3y + 70x^2y^2 - 42xy^3$
9. (a) $\frac{1}{2}a^2 - \frac{1}{2}ab + \frac{1}{8}a$ (b) $\frac{c^2}{2} - \frac{a}{4} + \frac{3c^3}{4}$
10. (a) $\frac{a^2}{3} - \frac{ab^2}{9} + \frac{a}{27}$
11. Express each of the following using symbols and brackets:
    (a) The product of two integers which differ by 3.
    (b) Multiply the sum of b and c by a.
    (c) Multiply h + 3 by h − 3.
    (d) Subtract a + b from 1 − b.
12. Find the area of the rectangle given below. The dimensions are in centimetres:

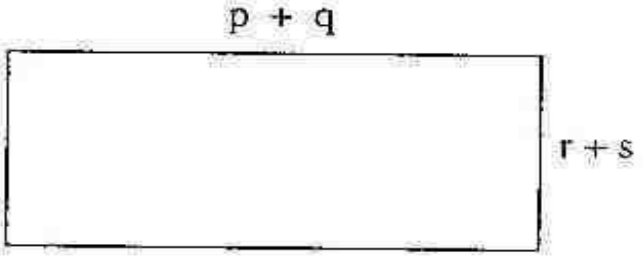

Fig. 10.3

13. Find the perimeter and the area of the figure 10.4:

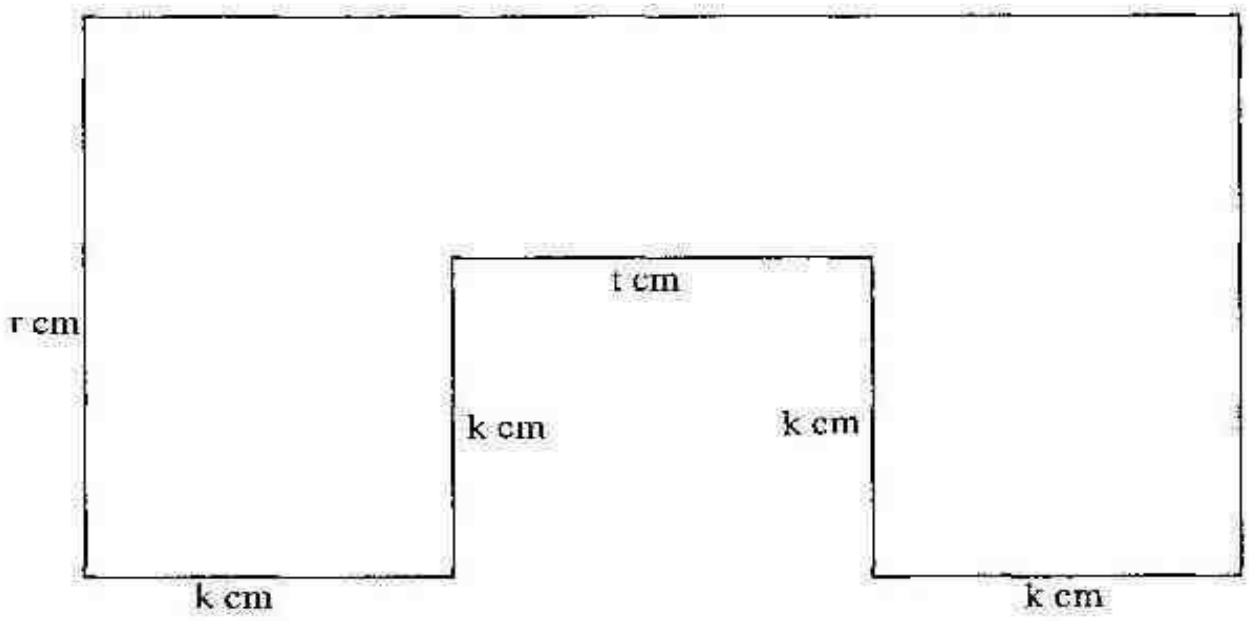

Fig. 10.4

14. An empty basket weighs w kg. Thirty eggs, each weighing p grams, are put in the basket. Find the mass of q such baskets.
15. Charo has sh. p. She buys a book for sh. q and a pen for sh. r. How much is she left with?
16. A man earns p shillings and his wife earns q shillings. They spend $\frac{1}{4}$ of their total earnings on food. How much are they left with?
17. A rectangular piece of cloth is $(x + 5)$ cm by $(x - 1)$ cm. A strip 2 cm wide is cut off all around it. Write an expression for the area of the strip.
18. A square lawn of side y m is surrounded by a path of width 1 m. Write an expression for the area of the path.
19. A car travels at a speed of $(x + 6)$ km/h. What distance does it cover in $(2y - 3)$ hours?
20. George has x books while Kilonzo has three more books than him. Kezia has t books and Edna one less than Kezia. How many books do they have altogether?
21. A father is three times as old as his son. Find an expression for the product of their ages five years ago, if the son is x years old now.
22. In figure 10.5, the outer radius is R cm while the inner radius is r cm. Find an expression for the area of the shaded region. (Leave your answer in terms of π)

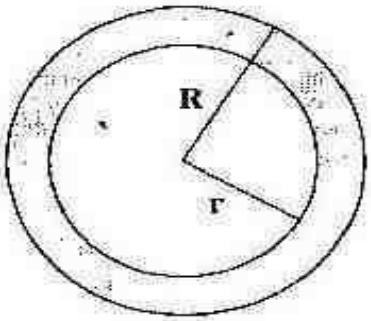

Fig. 10.5

10.4: Factorisation by Grouping

In the previous section, we dealt with factorisation of such expressions as $bx + b = b(x + 1)$. Factorise the following expressions:

(i) $2x + 6c^2$

(ii) $7y^2 + 14y + 21$

(iii) $3mn + 9n + 12n^2$

(iv) $ax + b + a + bx$

Note:

In (i), (ii) and (iii), there is a common factor while in (iv) there is no common factor. If the terms of the expression $ax + b + a + bx$ are taken pairwise, i.e., $ax + a$ and $bx + b$, then each pair has a common factor.

We can thus factorise the expression $ax + a + bx + b$ as; $a(x + 1) + b(x + 1)$. Now, $x + 1$ is a common factor. Therefore;

$$ax + b + a + bx = a(x + 1) + b(x + 1)$$
$$= (x + 1)(a + b)$$

This method is known as **factorisation** by grouping.

Note that the terms of this expression could be paired differently to obtain the same result. Find out this by factorising the expression anew.

Example 9

Factorise: (a) $3ab + 2b + 3ca + 2c$ (b) $ab + bc - a - c$

Solution

(a) $3ab + 2b + 3ca + 2c = b(3a + 2) + c(3a + 2)$
$= (3a + 2)(b + c)$

(b) $ab + bc - a - c = b(a + c) - 1(a + c)$
$= (a + c)(b - 1)$

Exercise 10.6

Factorise each of the following expressions:

1. (a) $nx - 2n + 3mx - 6m$ (b) $x^2 + xy + 2x + 2y$

2. (a) $3n - 3w + mw - mn$ (b) $3ab - 3bc - 2c + 2a$
3. (a) $x^2 - xy + 4x - 4y$ (b) $2ab + abk - 2m - mk$
4. (a) $x^2 + xc + bx + bc$ (b) $xr - ym + yr - xm$
5. (a) $ay + 3 + y + 3a$ (b) $ef^2 + gf + ef + g$
6. $ar^2 + ap - 2r^2 - 2p$

10.5: Algebraic Fractions

In algebra, as in arithmetic, fractions can be added and subtracted by finding the L.C.M. of the denominators.

Example 10

Express each of the following as a single fraction:

(a) $\frac{x-1}{2} + \frac{x+2}{4} + \frac{x}{5}$ (b) $\frac{a+b}{b} - \frac{b-a}{a}$

(c) $\frac{1}{3(a+b)} + \frac{3}{8(a+b)} + \frac{5}{12a}$

Solution

(a) $\frac{x-1}{2} + \frac{x+2}{4} + \frac{x}{5} = \frac{10(x-1)+5(x+2)+4x}{20}$

$= \frac{19x}{20}$

(b) $\frac{a+b}{a} - \frac{b-a}{b} = \frac{b(a+b)-a(b-a)}{ab}$

$= \frac{a^2+b^2}{ab}$

(c) $\frac{1}{3(a+b)} + \frac{3}{8(a+b)} + \frac{5}{12a}$

The L.C.M. of 3, 8 and 12 is 24
The L.C.M. of a and (a + b) is a(a + b)
∴ The L.C.M. of 3(a + b), 8(a + b), and 12a is 24a(a + b)

$\frac{1}{3(a+b)} + \frac{3}{8(a+b)} + \frac{5}{12a} = \frac{8a+9a+10a+10b}{24a(a+b)}$

$= \frac{27a+10b}{24a(a+b)}$

Exercise 10.7

1. Find the L.C.M. of:
   (a) 2t, 3t, 5t (b) r, s, t
   (c) $2 \times 3, 2^2 \times 7, 3^2 \times 5$ (d) ab, a^2b, b^2a
   (e) $4ab^2$, $8a^2b$ (f) a(2b + c), b(2c + a), c(2a + b), 2ap, 2bp

Express each of the following as a single fraction in its lowest form:

2. (a) $\frac{x+1}{2}+\frac{x-1}{3}$ (b) $\frac{2a^2+ab}{ab}-\frac{3a^2-ab}{6ab}$

3. (a) $\frac{m}{3}+\frac{x-1}{2}+\frac{x}{6}$ (b) $\frac{3a^2+ab}{6(a+b)}+\frac{a}{3}$

4. (a) $\frac{p+q}{3}+\frac{p-2q}{5}$ (b) $\frac{1}{2}+\frac{3a+b}{2a+2b}$

5. (a) $\frac{2r-3}{4}+\frac{1-r}{3}$ (b) $\frac{2}{a^2}-\frac{1}{c^2d}$

6. (a) $\frac{3x-6}{4}-\frac{2x-2}{3}$ (b) $\frac{1}{cd^2}-\frac{1}{c^2d}$

7. (a) $\frac{1+v}{u}+\frac{1-u}{v}$ (b) $\frac{1}{a^2bc}+\frac{a+b^2}{ab^2c}+\frac{1}{abc^2}$

8. (a) $\frac{r+s}{r}+\frac{r+s}{s}$ (b) $\frac{1}{ab}+\frac{a+b}{a^2b+ab^2}$

9. (a) $\frac{p+q}{t}-\frac{p-q}{s}$ (b) $\frac{6ab-2ab^2}{2a+2ab}+\frac{8b}{2a+2b}$

10. (a) $\frac{p+q}{p}+\frac{p-q}{q}$ (b) $\frac{a^2b}{4ab}+\frac{b^2a}{4ab}+\frac{3}{4}$

11. (a) $\frac{2r+t}{t}+\frac{r+1}{r}$ (b) $\frac{4p^3r}{2pr-3r^2p}-\frac{2r^2p}{2pr-3r^2p}$

10.6: Simplification by Factorisation

One of the uses of factortisation is simplification of given expressions.

Example 11

Simplify:

(a) $\frac{ra+rb}{ma+mb}$ (b) $\frac{ax-ay+bx-by}{a+b}$ (c) $\frac{ay-ax}{bx-by}$

Solution

(a) $\frac{ra+rb}{ma+mb}=\frac{r(a+b)}{m(a+b)}$

$=\frac{r}{m}$

(b) $\frac{ax-ay+bx-by}{a+b} = \frac{a(x-y)+b(x-y)}{a+b}$

$= \frac{(a+b)(x-y)}{a+b}$

$= x - y$

(c) $\frac{ay-ax}{bx-by} = \frac{a(y-x)}{b(x-y)} = \frac{a(y-x)}{-b(y-x)} = \frac{a}{-b} = \frac{-a}{b}$

Note:

$x - y = -(y - x)$

Exercise 10.8

Simplify by use of common factors:

1. (a) $\frac{x^2-4x}{x-4}$ (b) $\frac{2a^2+a^3}{2a+a^2}$
2. (a) $\frac{3bx-3by+4ax-4ay}{4a+3b}$ (b) $\frac{3bx-3by+4ax-4ay}{x-y}$
3. (a) $\frac{18ar-18am}{9am-9ar}$ (b) $\frac{4xy-3x+8y^2-6y}{8y-6}$
4. (a) $\frac{2m-am-2y+ay}{2m+2y-am-ay}$ (b) $\frac{4xy-3x+8y^2-6y}{x+2y}$
5. (a) $\frac{x^2-4ax-4a+x}{(x+1)(4a^2-ax)}$ (b) $\frac{(x+1)\{a(x-1)+b(x-1)\}}{(1-x)(ax+bx-a-b)}$

10.7: Substitution

The process of giving variables specific values in an expression is known as **substitution.**

Consider the rectangle below. Its area is $(a + 3)(b + 2)$ cm^2:

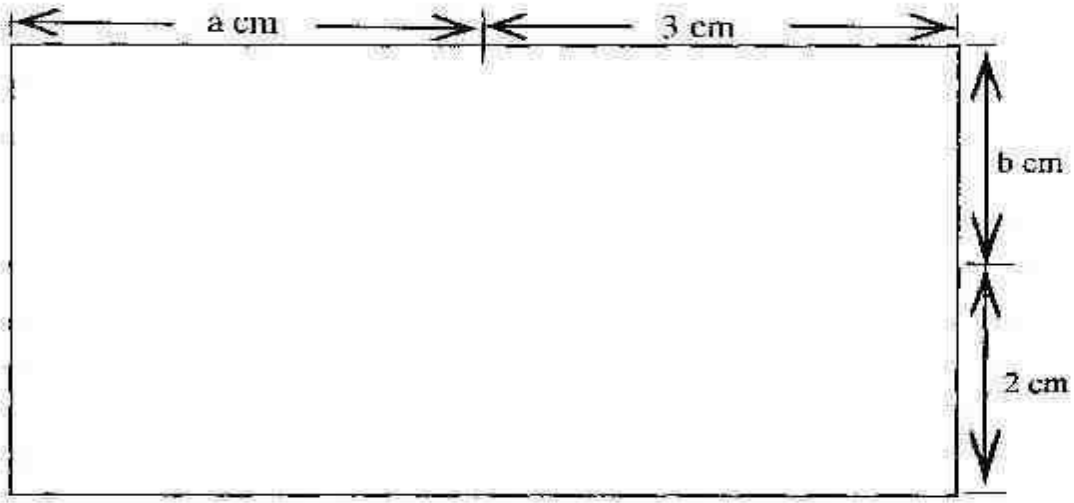

Fig. 10.6

If a = 4 and b = 1, then the area of the rectangle becomes $(4 + 3)(1 + 2) = 7 \times 3 = 21 \text{ cm}^2$

If a = 10 and b = 7 then the new area is $(10 + 3)(7 + 2) = 13 \times 9 = 117 \text{ cm}^2$

Example 12

Evaluate the expression $\frac{x^2 + y^2}{y + 2}$ if x = 2 and y = 1.

Solution

$$\frac{x^2 + y^2}{y + 2} = \frac{2^2 + 1^2}{1 + 2}$$

$$= \frac{4 + 1}{3}$$

$$= \frac{5}{3}$$

$$= 1\frac{2}{3}$$

Exercise 10.9

1. Evaluate:
   (a) $a^2 - b^3$ when: (i) a = 1, b = 2 (ii) a = 3, b = 1
   (iii) a = 0.2, b = 0.3
   (b) $a^2 - ab$ when: (i) a = 2, b = 1 (ii) a = 3, b = 0
   (iii) a = 3, b = 3 (iv) $a = \frac{1}{16}, b = \frac{1}{4}$
2. If r = 5, s = 2, and t = 3, find the value of:
   (a) $r^2 + s^2 - t$ (b) $\frac{r}{s} + \frac{s}{t} + \frac{t}{r}$
   (c) $\frac{r}{s+t} - \frac{s}{t+r} - \frac{t}{r+s}$ (d) $\frac{s+t}{r}$
   (e) $\frac{r+s}{t} + 2s$ (f) $\frac{2s - r}{s} - \frac{3}{5}t$
3. If $A = (R^2 - r^2)$, find A when:
   (a) R = 19 and r = 11.
   (b) R = 0.6 and r = 0.2.
4. If $E = \frac{1}{2}mv^2$, find m when E = 30 and v = 2.
5. If $V = \frac{1}{3}l \times b \times h$, find:
   (a) h if lb = 8 and V = 24.
   (b) b if lh = 12 and V = 96.
6. The surface area of an open box of side a, b and c centimetres is given by $A = 2b(a + c) + ac$.

(a) Find A if a = 40, b = 30 and c = 20.

(b) Find c if A = 512, a = 6 and b = 10.

Mixed Exercise 1

1. Simplify each of the following expressions:

(a) $\frac{1}{a} - \frac{a}{b} + \frac{3}{a} + \frac{4b}{a}$

(b) $a^2b + b^2a + 3ba^2 - 3b^2a + b^3a + a^3b$

(c) $\frac{\frac{1}{c} + \frac{1}{d}}{c+d}$

(d) $\frac{3a^3r}{4r^2a^2}$

(e) $\frac{a-3}{(3+a)(3-a)}$

(f) $\frac{3a}{2b} + \frac{4a}{3b} + \frac{5c}{4b}$

(g) $3q - 3 - (1 + q) + 1 - q - \frac{1}{q}$

(h) $\frac{x-2y}{12p} - \frac{x+3y}{60p}$

2. Three girls share an amount of money. The eldest gets $\frac{b}{3c}$ of the total amount while the youngest gets $\frac{b}{6c}$ of the total. What fraction does the third girl get?
3. Find the sum of a third of (a + b) and a fifth of (a − b).
4. After the tenth month of the year, what fraction of the year still remains?
5. A rectangular field is 0.4 m longer than it is wide. If its length is 6m find its perimeter. When the breadth of the rectangle is reduced by 0.5 m, the length is increased such that the perimeter is increased by $\frac{1}{4}$ of its original. What is the change in the length of the rectangle?
6. (a) Convert each of the following into a decimal:

(i) $\frac{7}{8}$ (ii) $1\frac{1}{8}$ (iii) $5\frac{3}{7}$ (iv) $\frac{11}{9}$

(b) Convert each of the following into a fraction:

(i) 0.375 (ii) 0.84 (iii) 2.4 (iv) 0.275

(c) Copy and complete each of the following:

(i) $\frac{3}{8} = \frac{}{2} = \frac{-15}{-40} = \frac{}{40}$

(ii) $\frac{13}{4} = \frac{5.2}{1.6} = \frac{0.65}{}$

7. The internal height of a rectangular box is 10.5 cm. The thickness of the bottom is $\frac{3}{5}$ cm and the thickness of the top is 1 cm. What is the external height of the box?

8. Three cisterns in a public lavatory are designed to flush at intervals of 8, 13 and 15 seconds. After how many minutes will they flush together?
9. Three-fifths of work is done on the first day. On the second day, $\frac{3}{4}$ of the remainder is completed. If third day $\frac{7}{8}$ of what remained is done, what fraction of work still remains to be done?
10. Juma spent half of his July salary on school fees, one-eighth on farming and two-thirds of the remainder on food. Calculate his July salary if he spent sh. 3200 on food.
11. A farmer has 3 containers of capacity 48 litres, 36 litres and 27 litres. Find the capacity of:
    (a) the smallest container that can be filled by each one of them an exact number of times.
    (b) the largest container that can be used to fill each one of them an exact number of times.
12. Onyango buys p oranges and discover that 1% are bad. How many oranges are fit for consumption?
13. Evaluate:

$$3\frac{7}{8} + \sqrt{\left(\frac{3\frac{7}{8} \div 7\frac{3}{4}}{5\frac{1}{10} + 2\frac{9}{10}}\right)}$$

14. Use tables of squares to evaluate:

    (a) $6250^2 \div 0.1750^2$ (b) $0.0225^2 \times 12800^2$

    (c) $\dfrac{(0.706 \times 20.5)^2}{32.2}$ (d) $\dfrac{23.5^2 \times 0.701^2}{3.4}$

15. Find the length of a square whose area is 0.0084 m^2.
16. A foreign government donated sh. 67.9 billion while the Kenya Government contributed sh. 200 million towards a project. Of the total amount sh. 10.8 million was used to remunerate experts, sh. 670 000 for the purchase of stationery and sh. 12.8 million for the acquisition of machinery. How much money remained unused? (Express your answer in words)